

**The Institute of Mathematical Sciences  
The Chinese University of Hong Kong**

Workshop on Wave Equations  
July 22, 2016 (Friday)

*Venue: Room 501a, Academic Building No. 1, CUHK*

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**Stable shock formation for nearly simple outgoing plane symmetric waves**

**Jonathan Luk**

University of Cambridge and Stanford University

Abstract: We consider a class of quasilinear wave equations violating the classical null condition and show that there exists an open set of data representing “nearly simple outgoing plane symmetric waves” which lead to shock formation. This may be viewed as an extension of Lax’s blow-up result for  $2 \times 2$  strictly hyperbolic genuinely nonlinear system, where we now consider solutions which are non-plane-symmetric perturbations of a distinguished subset of the solutions studied by Lax. The proof is based on a geometric framework introduced by Christodoulou and also, perhaps somewhat surprisingly, the good null structure of the equations. The result covers the compressible Euler equations in two spatial dimensions. This is a joint work with Speck, Holzegel and Wong.

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**On global regularity and scattering for geometric wave equations**

**Sung-Jin Oh**

University of California, Berkeley

Abstract: I will survey and present recent results concerning the question of global regularity and scattering for geometric wave equations, with particular emphasis in energy critical dimensions. Examples of such equations include the wave map, Maxwell-Klein-Gordon, Maxwell-Dirac and Yang-Mills equations.

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## **Decay properties of solutions of Maxwell Klein-Gordon equations**

**Shiwu Yang**

University of Cambridge

Abstract: In this talk, I will present a recent progress on the asymptotic behaviour of global solutions of Maxwell Klein-Gordon equations. We show that the integrated local energy and the energy flux through the outgoing null hypersurfaces decays polynomially in the retarded time in Minkowski space with data merely bounded in some gauge invariant weighted Sobolev space. This in particular includes the case with large charge. One novelty of this work is that these decay estimates precisely capture the asymptotic properties for the non-linear fields with arbitrarily large data. If in addition that the initial data for the scalar field is sufficiently small, then we show the pointwise decay of the solutions. This result improves the previous result of Lindblad-Sterbenz in which smallness is required for both the scalar field and the Maxwell field.

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## **On the global dynamics of three dimensional incompressible magnetohydrodynamics**

**Pin Yu**

Tsinghua University

Abstract: We construct and study global solutions for the 3-dimensional incompressible MHD systems with arbitrary small viscosity. In particular, we provide a rigorous justification for the following dynamical phenomenon observed in many contexts: the solution at the beginning behave like non-dispersive waves and the shape of the solution persists for a very long time (proportional to the Reynolds number); thereafter, the solution will be damped due to the long-time accumulation of the diffusive effects; eventually, the total energy of the system becomes extremely small compared to the viscosity so that the diffusion takes over and the solution afterwards decays fast in time. We do not assume any symmetry condition. The size of data and the a priori estimates do not depend on viscosity. The proof is built upon a novel use of the basic energy identity and a geometric study of the characteristic hypersurfaces. The approach is partly inspired by Christodoulou-Klainerman's proof of the nonlinear stability of Minkowski space in general relativity. This is a joint work with Ling-Bing HE (Tsinghua University) and Li XU (Chinese Academy of Sciences).